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Ground-State and Thermal Properties of a Long-Range Josephson Array

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The ground-state and thermal properties of a two-dimensional array of Josephson coupled superconducting wires is studied. For rational values of the magnetic flux per plaquette, mean-field theory provides an accurate description and the system makes a transition to a vortex state. For small values of the magnetic flux per plaquette, mean-field theory also provides an adequate description. The measurement of critical currents can provide information on the ground state.

KEY WORDS: Josephson array; phase transition.

1. INTRODUCTION

In this paper we study the ground-state and thermal properties of a model proposed by Vinokur *et al.*[1] This model consists of N vertical and N horizontal superconducting filaments or wires arranged in two parallel planes in such a way that each vertical filament is connected by Josephson junctions to each horizontal filament and vice versa. A perpendicular magnetic field is applied to the planes. The energy E_0 of an individual Josephson junction is much smaller than the transition temperature of a filament. We neglect induced fields due to currents in the junctions and in the filaments, since they are much smaller than the external magnetic field. Also, at temperatures not close to the superconducting transition of an individual wire, fluctuations of the modulus and phase of the order parameter in the wire can be neglected. Each wire is then described by its phase and we denote the phase of the vertical and horizontal wires by ϕ_k and ψ_j, respectively ($k, j = 1,..., N$). As the junctions are weak, the current flowing in any individual wire is small and its effects are neglected.

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Geometrically, this model, in which each vertical wire interacts with each horizontal wire and vice versa, has some features of an infinite-range model. Vinokur *et al.*[1] and Chandra *et al.*[3] have studied the disordered array, in which the distance between neighboring wires is assumed to be random, and have shown that it behaves like a vector spin glass. In this paper we study the case of a uniform array and show that it is analytically solvable in certain cases. The uniform array has also been studied by Chandra *et al.*,[3] but for a different range of the flux.

With the assumptions above, the energy of the array takes the form

$$E(2\pi/\alpha) = -E_0 \sum_{j,k=0}^{N-1} \cos(\phi_k - \psi_j + \alpha k_j) \tag{1}$$

where $\alpha = 2\pi H a^2/\varphi_0$, H is the external magnetic field, a is the wire spacing, and φ_0 is the flux quantum. This Hamiltonian is analogous to that of an XY model in which the interaction is modulated by the magnetic field.

2. SYMMETRY PROPERTIES

We consider the symmetry properties of the model in the case $\alpha = 2\pi/n$ ($n < N$). In order to avoid edge effects, we use periodic boundary conditions and suppose that n is a divisor of N. The energy is then invariant under the transformation

$$\phi_k \to \phi_k + \frac{2\pi mk}{n}, \quad \psi_j \to \psi_{j+m} \qquad \text{or} \qquad \psi_j \to \psi_j - \frac{2\pi mj}{n}, \quad \phi_k \to \phi_{k+m} \tag{2}$$

where m is an integer and we only need to consider $m = 1,..., n-1$. Under this transformation a correlation function

$$\langle e^{i(\phi_k - \phi_l)} \rangle = e^{2\pi i m(k-l)/n} \langle e^{i(\phi_k - \phi_l)} \rangle \tag{3}$$

where the angular brackets indicate a thermal average with Boltzmann factor $e^{-\beta E(n)}$. Thus, we conclude that the correlation functions

$$\langle e^{i(\phi_k - \phi_l)} \rangle = A(n, T)\, \delta_{k,l} \pmod n \tag{4}$$

with a similar result for the ψ variables. Here $A(n, T)$ is a factor depending on n and temperature. In particular, for $n = N$, all the ϕ (or ψ) variables are uncorrelated. Higher order correlation functions, such as $\langle \exp i(\phi_{k_1} + \phi_{k_2} - \phi_{k_3} - \phi_{k_4}) \rangle$, will vanish unless $k_1 + k_2 - k_3 - k_4 = 0 \pmod n$.

For the mixed correlation functions of the ϕ and ψ variables, application of (2) gives

$$\begin{aligned} F_{kj} \equiv \langle e^{i(\phi_k - \psi_j)} \rangle &= e^{2\pi i m_1 k/n} \langle e^{i(\phi_k - \psi_{j+m_1})} \rangle \\ &= e^{2\pi i m_2 j/n} \langle e^{i(\phi_{k+m_2} - \psi_j)} \rangle \\ &= e^{2\pi i (m_1 k + m_2 j + m_1 m_2)/n} \langle e^{i(\phi_{k+m_2} - \psi_{j+m_1})} \rangle \end{aligned} \tag{5}$$

These relations enable us to relate all the pair correlation functions to a single one, say F_{00}. Thus,

$$F_{kj} = e^{-2\pi i k j/n} F_{00} \tag{6}$$

The energy is given by

$$\begin{aligned} \langle E \rangle &= -E_0 \operatorname{Re} \sum_{kj} e^{2\pi i k j/n} F_{kj} \\ &= -E_0 N^2 \operatorname{Re} F_{00} \end{aligned} \tag{7}$$

These symmetry properties are broken in the ordered state, as we show below.

3. GROUND-STATE ENERGY

We can write the energy (1) in the form

$$E(n) = -\frac{E_0 N}{2} \sum_{j=0}^{N-1} (e^{-i\psi_j} g_j + \text{c.c.}) \tag{8}$$

where

$$g_j = \frac{1}{N} \sum_{k=0}^{N-1} e^{i\phi_k + 2\pi i j k/n} \tag{9}$$

is an effective field, acting on the ψ variables due to the ϕ variables. The energy is minimized by choosing $\psi_j = \arg g_j$, so that

$$E_G(n) = -E_0 N \sum_{j=0}^{N-1} |g_j| \tag{10}$$

There are only n distinct values of g_j with $j = 1, \ldots, n-1$ so that (we suppose n is a divisor of N)

$$E_G(n) = -E_0 \frac{N^2}{n} \sum_{j=0}^{n-1} |g_j| \tag{11}$$

We must now choose the ϕ_k to minimize this energy. For $n=1$ it is trivial and $E_G(1) = -E_0 N^2$. As an example we consider $n=2$. We can write

$$g_0 = \tfrac{1}{2}(h_0 + h_1), \qquad g_1 = \tfrac{1}{2}(h_0 - h_1) \tag{12}$$

where

$$h_0 = \frac{2}{N} \sum_{k\,(\mathrm{even})} e^{i\phi_k}, \qquad h_1 = \frac{2}{N} \sum_{k\,(\mathrm{odd})} e^{i\phi_k}$$

and

$$E_G(2) = -\tfrac{1}{4} E_0 N^2 (|h_0 + h_1| + |h_0 - h_1|) \tag{13}$$

The energy is minimized with respect to the phases and magnitudes of h_0 and h_1. The minimum occurs when they differ in phase by $\pi/2$ and $|h_0| = |h_1| = 1$, giving

$$E_G(2) = -E_0 N^2/\sqrt{2} \tag{14}$$

This procedure can be generalized for any integer n by writing

$$g_j = \frac{1}{n} \sum_{k=0}^{n-1} h_k e^{2\pi ijk/n} \tag{15}$$

where

$$h_k = \frac{n}{N} \sum_{l=0}^{N/n-1} e^{i\phi_{k+nl}}, \qquad k = 0,..., n-1 \tag{16}$$

Again we minimize with respect to the phase and magnitude of h_k. The minimum is obtained when all the $|g_j|$ are equal and the h_k are "perpendicular":

$$\sum_{k=0}^{n-1} h_k h^*_{k-m} = 0, \qquad m = 1,..., n-1 \tag{17}$$

Then $|g_j| = (1/n)(\sum_{k=0}^{n-1} |h_k|^2)^{1/2} = n^{-1/2}$ and

$$E_G(n) = -E_0 N^2/\sqrt{n} \tag{18}$$

To get a finite energy per wire we need $E_0 \propto 1/N$. In particular, for $n=N$ the ground-state energy is

$$E_G(N) = -E_0 N^{3/2} \tag{19}$$

which requires $E_0 \sim 1/N^{1/2}$ in order that the energy/wire be finite. A solution of (17) is

$$\begin{aligned} h_k &= e^{\pi i k^2/n + i\theta_0} \qquad n \text{ even} \\ &= e^{2\pi i k^2/n + i\theta_0} \qquad n \text{ odd} \end{aligned} \tag{20}$$

This solution has period n and corresponds to

$$\begin{aligned} \phi_k &= \frac{\pi k^2}{n} + \theta_0, \qquad \psi_j = -\frac{\pi j^2}{n} + \theta_0 + \frac{\pi}{4} \pmod{2\pi} \qquad (n \text{ even}) \\ \phi_k &= \frac{2\pi k^2}{n} + \theta_0, \qquad \psi_j = -\frac{\pi j^2}{2n} + \theta_n + c_{jn} \pmod{2\pi} \qquad (n \text{ odd}) \end{aligned} \tag{21}$$

where $c_{jn} = 0$ $(\pi n/2)$ for j even (odd), and where $\theta_n = \theta_0$, $\theta_0 + \pi/2$ for $n = 1, 5,...$ or $n = 3, 7,....$ The phase increases from one wire to the next. We can visualize the solution as a series of Josephson vortices, each vortex corresponding to a change in phase by 2π. Both solutions are periodic in n. In (21) we can replace $k \to k + m$ with $m = 0,..., n-1$ giving a degeneracy of at least n.

Properties of the ground state can be obtained by measuring critical currents. Suppose a current I enters each vertical wire and the same current I exits from each horizontal wire. We then solve

$$j_0 \sum_j \sin\left(\phi_k - \psi_j + \frac{2\pi k j}{n}\right) = I \tag{22}$$

with a similar equation (with $-I$) for the horizontal wires. A solution is

$$\phi_k = \frac{\pi k^2}{n} + \theta_0, \qquad \psi_j = \frac{-\pi j^2}{n} + \theta_0 - \frac{\pi}{4} \tag{23}$$

i.e., the relative phase $\phi_k - \psi_j$ is changed by $\pi/2$ from (21). The critical current is $I_c = j_0 N/n^{1/2}$.

4. THERMAL PROPERTIES

The partition function is given by

$$Z = \iint \left(\frac{d\phi}{2\pi}\right)\left(\frac{d\psi}{2\pi}\right) e^{-\beta E(n)} \tag{24}$$

and writing $E(n)$ in the form (8), we can integrate over the ψ variables, giving

$$Z=\int\left(\frac{d\phi}{2\pi}\right)\exp\left[\sum_{j=0}^{N}\ln I_0(x\,|g_j|)\right] \tag{25}$$

where $x=\beta E_0 N$ and the effective field g_j is given by (9). As there are only n different values of g_j, (25) becomes

$$Z=\int\left(\frac{d\phi}{2\pi}\right)\exp\left[\frac{N}{n}\sum_{j=0}^{n=1}\ln I_0(x\,|g_j|)\right] \tag{26}$$

In the cases where $\lim(N/n)=\infty$ this integral can be evaluated by stationary phase. It is convenient to first transform to an integral over the g_j by including a factor

$$\prod_{j=0}^{n-1} N^2\int d^2g_j\,\delta\left(Ng_{jx}-\sum_{k=1}^{N}\cos\left(\phi_k+\frac{2\pi jk}{n}\right)\right)$$
$$\times\,\delta\left(Ng_{jy}-\sum_{k=1}^{N}\sin\left(\phi_k+\frac{2\pi jk}{n}\right)\right) \tag{27}$$

After exponentiating the delta functions using

$$\delta(x)=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}e^{kx}\,dk \tag{28}$$

we can carry out the integrals over the ϕ_k. A change of variables from the k_j of (28) to

$$h_k=\sum_{j=0}^{n-1}k_j e^{-2\pi ijk/n} \tag{29}$$

brings Z into the convenient form

$$Z=N^{2n}\int(d^2g\,d^2h)\exp\left(\frac{N}{n}\left\{\sum_{j=0}^{n-1}\ln\left[I_0(x\,|g_j|)\,I_0(|h_j|)\right]\right.\right.$$
$$\left.\left.-\frac{1}{2}\sum_{k,j=0}^{n-l}(h_k^* e^{-2\pi ijk/n}g_i+\text{c.c})\right\}\right) \tag{30}$$

It is now suitable for evaluation by stationary phase. This leads to the equations

$$\frac{xI_1(x|g_j|)}{I_0(x|g_j|)}\frac{g_j}{|g_j|}=\sum_{k=0}^{n-1} e^{2\pi ijk/n}h_k \tag{31}$$

$$\frac{I_1(|h_k|)}{I_0(|h_k|)}\frac{h_k}{|h_k|}=\sum_{j=0}^{n-1} e^{-2\pi ijk/n}g_j \tag{32}$$

The critical temperature is obtained by linearizing these equations

$$\frac{x^2}{2}g_j=\sum_{k=0}^{n-1} e^{2\pi ijk/n}h_k, \qquad \frac{1}{2}h_k=\sum_{j=0}^{n-1} e^{2\pi ijk/n}g_j \tag{33}$$

which gives $x^2/4=n$ or

$$kT_c=\frac{E_0N}{2\sqrt{n}} \tag{34}$$

A finite T_c requires $E_0\sim 1/N$.

The free energy in the disordered phase above T_c is obtained by expanding the argument of the exponential in (30) keeping only quadratic terms. After choosing the contour for the g and h variables appropriately we find for the free energy

$$F_{\text{lin}}=kTn\log\left(1-\frac{T_c^2}{T^2}\right)+\text{const} \tag{35}$$

The stationary-phase procedure is only justified when

$$\lim_{N\to\infty}\frac{n}{N}=0$$

Nevertheless we might expect that the above mean-field theory would give qualitatively reasonable results in other cases. In particular for $n=N$, $kT_c=E_0\sqrt{N}/2$ and the fluctuation free energy (35) is proportional to $E_0N^{3/2}$.

Another case of interest is $\alpha=2\pi\xi$, where ξ is an irrational number. We can approximate $\xi\sim p/n$ when p and n are mutually prime integers which both tend to infinity. The above results for $\alpha=2\pi/n$ are also correct for $\alpha=2\pi p/n$, but require $n<N$. Qualitatively we see that the ground-state

energy (18) and transition temperature (34) decrease in magnitude as n increases and we approach an irrational number. However, the limit $n \to \infty$, $p \to \infty$ is not justified.

For irrational ξ the transformation (29) cannot be used and the linearized mean-field equations take the form

$$\frac{x^2}{2} g_j = N k_j, \qquad g_j = \frac{1}{2N} \sum_{k,j=0}^{N-1} e^{2\pi i \xi k(j-j')} k_{j'} \tag{36}$$

which leads to the equation

$$\lambda \bar{g}_j = \sum_{j'=0}^{N-1} \left(\frac{\sin \pi\xi N(j-j')}{\sin \pi\xi(j-j')} \right) \bar{g}_{j'} \tag{37}$$

where we have put $\lambda = 4(kT_c/E_0)^2$. We require the maximum eigenvalue of this equation. A limit can be placed on this by using a variational method,

$$\lambda = \sum_{j,j'=0}^{N-1} \bar{g}_j \left(\frac{\sin \pi\xi N(j-j')}{\sin \pi\xi(j-j')} \right) \bar{g}_{j'} \Big/ \sum_j \bar{g}_j^2 \tag{38}$$

As a variational solution, choose $\bar{g}_j = 1$ for one value of j and zero otherwise. Then $\lambda_{\max} \geqslant N$, $kT_c > \frac{1}{2} E_0 N^{1/2}$, demonstrating the existence of a transition (in the mean-field approximation if $E_0 \sim N^{-1/2}$).

5. OSCILLATIONS OF THE GROUND STATE

We assign a capacitance C_0 and ohmic conductivity σ_0 to each junction. The phase variables then satisfy the time-dependent equations

$$\ddot{\phi}_k - \ddot{\bar{\psi}} + \gamma(\dot{\phi}_k - \dot{\bar{\psi}}) + \frac{\omega_0^2}{N} \sum_{j=0}^{N-1} \sin(\phi_k - \psi_j + \alpha k_j) = I_k \tag{39}$$

$$\ddot{\psi}_j - \ddot{\bar{\phi}} + \gamma(\dot{\psi}_j - \dot{\bar{\phi}}) - \frac{\omega_0^2}{N} \sum_{k=0}^{N-1} \sin(\phi_k - \psi_j + \alpha k_j) = -J_j \tag{40}$$

where $\gamma = \sigma_0 / C_0$ and $\omega_0^2 = 4e^2 E_0 / h^2 C_0$ (ω_0 is the Josephson plasma frequency of a single junction). $\bar{\phi}$ and $\bar{\psi}$ are the average phases

$$\bar{\phi} = \frac{1}{N} \sum_{k=0}^{N=1} \phi_k, \qquad \bar{\psi} = \frac{1}{N} \sum_{j=0}^{N-1} \psi_j \tag{41}$$

and for current conservation

$$\sum_k I_k = \sum_j J_j$$

To investigate the plasma frequencies in the case $\alpha = 2\pi/n$ we set $I_k = J_j = 0$ and linearize these equations around the ground state by setting $\phi_k = \phi_k^0 + \phi_k'$, $\psi_j = \psi_j^0 + \psi_j'$, where the index zero indicates the ground-state configuration. Then, using (21), we get

$$\begin{aligned} \left(\left(\frac{\partial^2}{\partial t^2} + \gamma \frac{\partial}{\partial t} + \frac{\omega_0^2}{n^{1/2}}\right)(\phi_k' - \bar{\psi}'\right) = 0 \\ \left(\frac{\partial^2}{\partial t^2} + \gamma \frac{\partial}{\partial t} + \frac{\omega_0^2}{n^{1/2}}\right)(\psi_j' - \bar{\phi}) = 0 \end{aligned} \tag{42}$$

The solutions of these equations are all of the form

$$\phi_k' = a e^{2\pi i k/N - i\omega t}, \qquad \psi_j' = b e^{2\pi i j/N - i\omega t} \tag{43}$$

with dispersionless modes satisfying

$$\omega^2 - i\omega\gamma - \omega_0^2/n^{1/2} = 0 \tag{44}$$

which gives a reduction in the plasma frequency by a factor $n^{1/4}$.

6. SLOWLY VARYING PHASE

We can also obtain results in the case that α is small and the phases ϕ_k and ψ_j vary slowly from wire to wire in the ordered state. This requires that $\alpha N \ll 1$. The energy is written in the two forms

$$\begin{aligned} E &= -\frac{E_0}{2} \sum_{k=0}^{N-1} (e^{-i\phi_k} \hat{f}_k + \text{c.c}) \\ &= -\frac{E_0}{2} \sum_{j=0}^{N-1} (e^{-i\psi_j} \hat{g}_j + \text{c.c}) \end{aligned} \tag{45}$$

$$\begin{aligned} \hat{f}_k &= \sum_{j=0}^{N-1} e^{i\psi_j - i\alpha k_j} \\ \hat{g}_j &= \sum_{k=0}^{j=0} e^{i\phi_k + i\alpha k_j} \end{aligned} \tag{46}$$

When $\alpha N < 1$ and the phase is slowly varying we can replace sums by integrals. $\hat{f}_k$ and $\hat{g}_j$ in (45) are then the sum of a large number of terms, fluctuations should be small, and we can replace them by averages. We then get mean-field Hamiltonians for the ϕ and ψ variables

$$E_\phi = -\frac{E_0}{2} \sum_k (e^{-i\phi_k} f_k + \text{c.c})$$
$$E_\psi = -\frac{E_0}{2} \sum_j (e^{-i\psi_j} g_j + \text{c.c}) \tag{47}$$

where f and g are averages which we determine self-consistently from the equations

$$f_k = \frac{\int (d\psi) \exp(-\beta E_\psi \hat{f}_k)}{\int (d\psi) \exp(-\beta E_\psi)}, \qquad g_j = \frac{\int (d\phi) \exp(-\beta E_\phi \hat{g}_j)}{\int d\phi \exp(-\beta E_\phi)} \tag{48}$$

Carrying out the integrals, we get

$$f_k = \sum_j \frac{I_1(\delta |g_j|)}{I_0(\delta |g_j|)} \frac{g_j}{|g_j|} e^{-i\alpha k_j} \tag{49}$$

$$g_j = \sum_k \frac{I_1(\delta |f_k|)}{I_0(\delta |f_k|)} \frac{f_k}{|f_k|} e^{i\alpha k_j} \tag{50}$$

where $\delta = \beta E_0$.

When $\alpha N < 1$ we can replace the sums by integrals with $f_k \equiv f(x)$ and $g_i \equiv g(y)$, giving

$$f(x) = \frac{1}{a} \int_0^L dy \frac{I_1(\delta |g(y)|)}{I_0(\delta |g(y)|)} \frac{g(y)}{|g(y)|} e^{-i\alpha xy/a^2} \tag{51}$$

with a similar equation for $g(y)$. Here a is the wire spacing. A solution to these equations is provided by

$$f(x) = \Delta e^{-i\alpha x^2/2a^2 + i\gamma}, \qquad g(y) = \Delta e^{i\alpha y^2/2a^2 + i\beta} \tag{52}$$

Edge effects can be neglected if $\alpha N^2 > 1$. This is a solution provided

$$\Delta = \left(\frac{2\pi}{\alpha}\right)^{1/2} \frac{I_1(\delta\Delta)}{I_0(\delta\Delta)}, \qquad \beta = \gamma - \frac{\pi}{4} \pmod{2\pi} \tag{53}$$

Linearizing the first equation given

$$kT_c = E_0 \left(\frac{\pi}{2\alpha}\right)^{1/2} \tag{54}$$

and for $T < T_c$

$$\Delta^2 = 8\left(\frac{kT_c}{E_0}\right)^2 \left(1 - \frac{T}{T_c}\right) \tag{55}$$

At $T = 0$, $\Delta = (2\pi/\alpha)^{1/2}$ and in the continuum approximation the phases are

$$\begin{aligned} \phi(x) &= -\frac{1}{2}\alpha\frac{x^2}{a^2} + \beta + \frac{\pi}{4} \\ \psi(y) &= \frac{1}{2}\alpha\frac{y^2}{a^2} + \gamma - \frac{\pi}{4} \end{aligned} \tag{56}$$

The ground-state energy is

$$E_G = -E_0 N \left(\frac{2\pi}{\alpha}\right)^{1/2} \tag{57}$$

where $\gamma - \beta - \pi/4 = 0 \pmod{2\pi}$. For $\gamma - \beta - \pi/4 = \pi \pmod{2\pi}$ the energy is a maximum and the critical current is $I_c = j_0(2\pi/\alpha)^{1/2}$. The phases again give a ground state in which we have an array of Josephson vortices. These results break down if $\alpha < 1/N^2$, as the number of vortices in the system goes to zero. This leads to an estimate of the lower critical field for the appearance of vortices as $H_{c1} = \phi_0/N^2a^2$.

7. CONCLUSION

We have studied the thermal and ground-state properties of a regular 2D array of $2N$ superconducting wires in a perpendicular magnetic field coupled by Josephson junctions. The order parameter phase is assumed constant along the length of a wire—a model proposed by Vinokur *et al.*[1] The important parameter is the magnetic flux per plaquette $\alpha/(2\pi)$ measured in units of the flux quantum. Fluctuations are negligible when $\alpha = 2\pi p/n$ (p, n relatively prime integers with $n < N$) and for $\alpha \ll 1/N$ and the model is solvable by mean-field methods. If the coupling energy is scaled appropriately with N (the scaling depends on α), the model shows a mean-field transition to a vortex state. Fluctuations appear not to be negligible for $\alpha \sim 2\pi/N$. It may be possible to use our results to interpolate between $\alpha = 2\pi p/n$ and $\alpha \ll 1/N$ to obtain qualitative results in this region. Thus the formulas for the transition temperature (34) and (54) and the ground-state energy (18) and (57) match at $\alpha = 2\pi/N$. Our analytic results are confined to the above special values of $\alpha/(2\pi)$ and do not give information on the behavior when this quantity is irrational or when there is a small amount of disorder in the system. The mean-field transition temperature (34) for rational values of $\alpha/(2\pi)$ is larger than that estimated for irrational values [see below (38)] by the large factor $(N/n)^{1/2}$. This suggests that there may be a strong preference for the system to lock onto rational values.

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